3.3: ELECTROMAGNETIC DIFFRACTION BY A PLANAR ARRAY OF CIRCULAR DISKS*

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In a recent paper [1] the electromagnetic diffraction by a perfectly conducting circular disk was calculated. The induced surface current density was obtained as a power series in (ka), with $k = \frac{2\pi}{\lambda}$ = wave number and a = disk radius. These results are now used to calculate the diffraction by a planar rectangular array of disks. Problems of this sort are important in the studies of artificial dielectrics where the molecular dipoles of real dielectrics are replaced by conductors distributed regularly or at random in a supporting medium. For many cases, good approximate solutions have been found usually for conductors with dimensions very small compared to the wavelength or for some very simple geometrical configurations. The case of an array of disks has been the object of early investigations. In a first approximation, the disks are replaced by the induced electric and magnetic dipole moments which Bethe obtained during his studies of the diffraction by small holes [2]. Later, the so-called static interaction, where phase differences between the oscillating dipoles are neglected, was taken into account.

In the present paper the formalism of the above-mentioned paper [1] is used to calculate the effect of higher order multipole moments and the dynamic interaction between the induced dipole moments of an array of circular disks, where it is recognized that the interaction fields are dipole fields.

It is desirable to relate the theoretical results to quantities which can be easily checked experimentally. As for any approximation, our results hold only for certain ranges of the parameters, in our case for small disks where $ka < 1$. Free space problems are usually not very amenable to experimental checks. It is therefore proposed to investigate waveguide configuration which, by suitable arrangement, will turn out to be closely related to the free space problems [3].

Consider an arrangement of disks in a transverse plane of a rectangular waveguide as shown in Figure 1. The disks are positioned such that their multiple images with respect to the guide walls form a planar rectangular array with spacings c and d. Similarly the TE_{10} mode corresponds to two symmetrical plane waves incident at angles $\theta_i = \pm \sin^{-1} \frac{\lambda}{2g}$ and polarized perpendicularly to the plane of incidence. The

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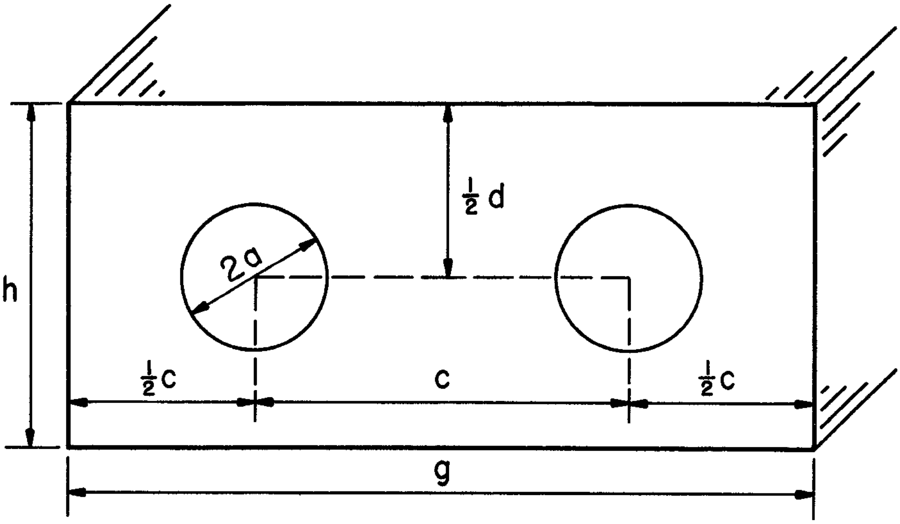

Fig. 1. Disk loaded rectangular waveguide.

reflected field is a TE_{10} mode only, traveling in the negative z-direction, provided that the size of the guide is chosen such that all other modes are cut off. That means that in the free space problem, the disk array represents a perfect partially transparent reflector.

The reflected field can be found from the induced currents in the disks. Using the Lorentz reciprocity principle we can find the reflection coefficient for each mode.

In a first approximation, Bethe's expression for the electric and magnetic dipole moments are used. The three cases for no interaction, static interaction (where it is assumed that the separation distances are small compared to the wavelength, so that propagation effects can be neglected) and dynamic interaction were considered. In the latter case, the dipole fields radiated from each disk are approximated by plane waves. The error involved should be small, if the separation between the disks is large compared to the wavelength. The doubly infinite sums which have to be evaluated are extremely slowly convergent. Calculation with a digital computer shows that several thousand terms are needed in order to insure some kind of accuracy. By a Fourier Transform series of this type it can, however, be converted into very rapidly converging series [4].

In a further approximation, higher order terms in the surface current distribution on the disks were evaluated. This made it possible to calculate higher order multipole moments. The expressions for the interaction between the disks were further improved by considering the actual dipole fields instead of a plane wave approximation. In a disk loaded rectangular waveguide, the reflected dominant mode can also be found by calculating an equivalent shunt susceptance at the position of the disks. Experimentally, this shunt susceptance can be easily measured which allows us to check the theoretical results.

Evaluation:

From the few numerical data available (theoretical and experimental), it is not possible to draw conclusions which are valid for every general case. The following evaluation has therefore a more qualitative character and the numerical values have to be taken with some reservation. The separation distances between the disks were of the order $(ks) \sim 2.5$.

Bethe Theory is applicable for	$(ka) < 0.5$
Third Order Theory is applicable for	$(ka) < 0.9$
Higher order multipoles have to be considered for	$(ka) > 0.6$
Static interaction is necessary for	$(ka) > 0.5$
Dynamic interaction in dipole field approximation is needed for	$(ka) > 0.7$.

The dynamic interaction in the plane wave approximation gives an improvement only if the separation of the disks is larger than the one considered here, probably for values of the order $(ks) > 6$.

1. W. H. Eggimann, "Higher Order Evaluation of Electromagnetic Diffraction by Circular Disks," Trans. IRE MTT-9, No. 5, 408-418 (1961).

2. H. A. Bethe, "Theory of Diffraction by Small Holes," Phys. Rev. 66, No. 2, 163-182 (1944).

3. R. E. Collin, "A Note on Waveguide Image Techniques," Case Institute of Technology, Sci. Rep. No. 19, AF 19(604)-3887 (November 1960).

4. R. E. Collin, W. H. Eggimann, "Dynamic Interaction Fields in a Two-Dimensional Lattice," Trans. IRE MTT-9, 110-115 (1961).